Upravte goniometrické výrazy:

1. $\frac{1-tg^2x}{\cos 2x} =$

   Riešenie:

$$\frac{1-tg^2x}{\cos 2x} = \frac{1-\frac{sin^2x}{cos^2x}}{cos^2x-sin^2x} =$$
$$= \frac{\frac{cos^2x-sin^2x}{cos^2x}}{cos^2x-sin^2x} = \frac{cos^2x-sin^2x}{cos^2x.(cos^2x-sin^2x)} = \frac{1}{cos^2x}$$

2. $\frac{cos^2x}{1+\sin x} =$

3. $\frac{1+tg^2x}{1+cotg^2x} =$

4. $\frac{\sin x-sin^3x}{\cos x-cos^3x} =$

5. $\frac{tg\ x}{1+tg^2x} =$

6. $1+\frac{cos^4x-1}{sin^2x} - cotg^2x.sin^2x =$

   Riešenie:

$$1+\frac{cos^4x-1}{sin^2x} - cotg^2x.sin^2x =$$
$$= 1+\frac{(cos^2x-1).(cos^2x+1)}{sin^2x} - \frac{cos^2x}{sin^2x}.sin^2x =$$
$$= sin^2x+cos^2x+\frac{(-sin^2x).(cos^2x+1)}{sin^2x} - cos^2x =$$
$$= sin^2x+cos^2x-(cos^2x+1)-cos^2x =$$
$$= sin^2x-cos^2x-1 =$$
$$= sin^2x-cos^2x-sin^2x-cos^2x =$$
$$= -2cos^2x$$

7. $1-sin^2x+cotg^2x.sin^2x =$
8. $\cos 2x+\sin 2x\,.tgx =$
9. $4\sin x\,.\cos x\,.(cos^2x-sin^2x) =$
10. $\left(\frac{1}{\cos x}+tg\ x\right).\left(\frac{1}{\cos x}-tg\ x\right) =$
11. $\frac{sin^2x}{\cos x}+\frac{tg\ x}{cotg\ x} =$

12. $\frac{tg^2\,x}{1+tg^2x}.\frac{1+cotg^2x}{cotg^2\,x}=$
13. $2\sin^2\frac{x}{2}+\cos x-2\cos^2\frac{x}{2}+\sqrt{1-sin^2x}=$
14. $\frac{\sin 2x}{1+\cos 2x}=$
15. $\frac{\sin 2x}{1-\cos 2x}=$
16. $\frac{1+\cos 2x}{1-\cos 2x}=$

Dokážte, že platí:

1) $\frac{tg^2\,x}{1+tg^2x}.\frac{1+cotg^2x}{cotg^2\,x}-tg^2x=0$
2) $cotg^2x-cos^2x=cotg^2x.cos^2x$
3) $\frac{cos^3x-sin^3x}{1+\sin x.\cos x}=\cos x-\sin x$
4) $1+\frac{2}{tg\,x+cotg\,x}=(\sin x+\cos x)^2$
5) $\sqrt{\frac{1+\sin x}{1-\sin x}}-\sqrt{\frac{1-\sin x}{1+\sin x}}=2.tg\,x$
6) $2-3cos^2x=3sin^2x-1$
7) $\frac{1}{cos^2x}-tg^2x=1$
8) $\frac{1}{cotg^2x}-tg^2x=1$